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a) i. $\dot{x} = 4\cos 2t$.

$\therefore x = \int \dot{x}\, dt$.

$= \int 4\cos 2t\, dt$

$= \left[4 \times \frac{1}{2} \sin 2t + c\right]$.

$\therefore 1 = 2\sin 2(0) + c$.

$\therefore 1 = c$.

$\therefore \dot{x} = 2\sin 2(t) + 1$.

ii. $\dot{x} = 0$.

$\therefore 0 = 2\sin 2t + 1$.

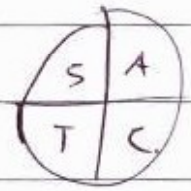
$-1 = 2\sin 2t$.

$-\frac{1}{2} = \sin 2t$.

$\therefore 2t = 210^\circ$.

$t = 105$.

$t = \frac{7\pi}{12}$ sec.



iii. $x = \int 2\sin 2t + 1$

$= 2 \times -\frac{1}{2}\cos 2t + t + c$.

$= -\cos 2t + t + c$.

when $x = 0$. $t = 0$.

$\therefore 0 = -(\cos 2(0)) + 0 + c$

$= +1 + c$.

$-1 = c$.

$\therefore x = -\cos 2t + t - 1$.

b. i. $y = x^2$

$y' = 2x$.

$\therefore m = 2(-1)$

$= -2$.

$\therefore y - y_1 = m(x - x_1)$.

$y - 1 = -2(x + 1)$.

$y - 1 = -2x - 1$.

$2x + y = 0$.

ii. Midpoint $(AB) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$

$= \left(\frac{-1 + 2}{2}, \frac{1 + 4}{2}\right)$

$= \left(\frac{1}{2}, \frac{5}{2}\right)$.

$m(AB) = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4 - 1}{2 + 1}$.

$= \frac{3}{3}$

$= 1$.

$m(C) = 1$.

$1 = 2x$. $C = \left(\frac{1}{2}, \frac{1}{4}\right)$

$\therefore x = \frac{1}{2}$.

Since $x = \frac{1}{2}$ at C.

and $M = \left(\frac{1}{2}, \frac{5}{2}\right)$.

$m(MC) = \frac{\frac{5}{2} - \frac{1}{4}}{\frac{1}{2} - \frac{1}{2}}$

$= 0$.

$\therefore$ MC is vertical.

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iii. MT $x = \frac{1}{2}$.

AT $2x + y = 0$.

$\therefore 2(\frac{1}{2} + y) = 0$.

$\therefore 1 + 2y = 0$.

$2y = -1$.

$y = -\frac{1}{2}$.

$2x - \frac{1}{2} = 0$.

$2x = \frac{1}{2}$.

$x = \frac{1}{4}$.

$\therefore$ co-ordinates (T) $-(-\frac{1}{2}, \frac{1}{4})$.

$\therefore \frac{y_2 - y_1}{x_2 - x_1} = m(BT)$.

$= \frac{4 - \frac{1}{4}}{2 + \frac{1}{2}}$

$= \frac{3.75}{2.5}$.

$\therefore m(BT) = \frac{3}{2}$. $\therefore y' = 2x$

$\therefore$ eqn ~~$y - y_1 = m(x - x_1)$~~ = 4.

~~$y - 4 = \frac{3}{2}(x - 2)$~~.

~~$2y - 8 = 3x - 6$~~

~~$2y - 2 = 3x$~~.

~~$\therefore 3x - 2y + 2 = 0$~~.

$\therefore$ tangent to parabola at (2, 4)

$\therefore$ [illegible]

You may ask for an extra Writing Booklet if you need more space to answer question 7.