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πr^2 = Area $2\pi r$ = circumference.

a) $A = 2\pi r^2 + 2\pi rh$

$V = 10\text{ m}^3$

$V = \pi r^2 \times h$

$10 = \pi r^2 h$

$\cancel{r^2 = \frac{10}{\pi h}}$

$\frac{10}{r^2} = \pi h$ — ①.

$A = 2\pi r^2 + 2\pi rh \to$ ②.

sub ① into ②.

$A = 2\pi r^2 + 2 \times \frac{10}{r^{\cancel{2}}} \times \cancel{r}$

$= 2\pi r^2 + \frac{20}{r}$ ✓

(ii) $\frac{dA}{dr} = 4\pi r \cancel{4} - 20r^{-2}$

$0 = 4\pi r - \frac{20}{r^2}$

$0 = 4\pi r^3 - 20$

$4\pi r^3 = 20$

$\pi r^3 = 5$

$r = \sqrt[3]{\frac{5}{\pi}}$

stat pt at $r = \sqrt[3]{\frac{5}{\pi}}$

$\frac{d^2A}{dr^2} = 4\pi + 40r^{-3}$ at $r = \sqrt[3]{\frac{5}{\pi}}$

$= 4\pi + \frac{40}{\left(\sqrt[3]{\frac{5}{\pi}}\right)^3}$

$= 4\pi + \frac{40}{\frac{5}{\pi}}$

$= 4\pi + \frac{\pi}{5} \times 40$

Since $\frac{d^2A}{dr^2} > 0$, $r = \sqrt[3]{\frac{5}{\pi}}$ is a minimum.

b)(i) $\sec^2 x + \sec x \tan x = \dfrac{1+\sin x}{\cos^2 x}$

$\dfrac{1}{\cos^2 x} + \dfrac{1}{\cos x} \times \dfrac{\sin x}{\cos x} = \dfrac{1+\sin x}{\cos^2 x}$

$\dfrac{1}{\cos^2 x} + \dfrac{\sin x}{\cos^2 x} = \dfrac{1+\sin x}{\cos^2 x}$

$\dfrac{1+\sin x}{\cos^2 x} = \dfrac{1+\sin x}{\cos^2 x}$ ✓

(ii) $\sec^2 x + \sec x \tan x = \cancel{\dfrac{1+\sin x}{\cos^2 x}}\ \dfrac{1}{1-\sin x}$

$\dfrac{1}{\cos^2 x} + \dfrac{\sin x}{\cos^2 x} = \dfrac{1}{1-\sin x}$

$1+\sin x = \dfrac{1}{1-\sin x} \times \cos^2 x$

$(1-\sin x)(1+\sin x) = \cos^2 x$

$1 - \sin x + \sin x - \sin^2 x = \cos^2 x$

$1 - \sin^2 x = \cos^2 x$

$1 = \sin^2 x + \cos^2 x$ ✓

$\therefore$ ~~the~~ $\sec^2 x + \sec x \tan x = \dfrac{1}{1-\sin x}$

(iii) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{1-\sin x}\,dx = \int_0^{\frac{\pi}{4}} \sec^2 x\,dx + \int_0^{\frac{\pi}{4}} \sec x \overset{\tan x}{\cancel{\tan x}}\,dx$

$= \Big[\tan x\Big]_0^{\frac{\pi}{4}} + \Big[\sec x\Big]_0^{\frac{\pi}{4}}$

$= \tan\frac{\pi}{4} - \tan 0 + \sec\frac{\pi}{4} - \sec 0$

$= 0.0137 - 0 + 1 - 1$

$= 0.0137.$

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c) $y = \frac{1}{x}$ for $x > 0$

$1 = \int_a^1 \frac{1}{x}\, dx$

$1 = \left[\log_e x\right]_a^1$

$1 = \log_e 1 - \log_e a$

$1 = 0 - \log_e a$

$1 = -\log_e a$

$e^1 = -a$

$e = -a$

$a = -e$

$1 = \int_1^b \frac{1}{x}\, dx$

$1 = \left[\log_e x\right]_1^b$

$1 = \log_e b - \log_e 1$

$1 = \log_e b - 0$

$e^1 = b$

$b = e.$

$\therefore$ $a = -e$ and $b = e$