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5 a i). area of circle = $\pi r^2$

volume of cylinder = $\pi r^2 \times h$

$10 = \pi r^2 \times h$

$h = \frac{10}{\pi r^2}$

$A = 2\pi r^2 + 2\pi r h$

sub $h = \frac{10}{\pi r^2}$ in ↑

$\therefore A = 2\pi r^2 + 2\pi r \left(\frac{10}{\pi r^2}\right)$

$= 2\pi r^2 + \frac{20\pi r}{\pi r^2}$

$= 2\pi r^2 + \frac{20}{r}$

ii). ~~stationary~~ minimum value occurs when $A' = 0$ & $A'' > 0$.

$A' = 4\pi r - 20r^{-2}$

$= 4\pi r - \frac{20}{r^2}$

$4\pi r - \frac{20}{r^2} = 0$

$\times r^2 \rightarrow 4\pi r^3 - 20 = 0$

$4(\pi r^3 - 5) = 0$

$\frac{4\pi r}{1} = \frac{20}{r^2}$

$r = \frac{20}{4\pi r^2}$

$r = \frac{5}{\pi r^2}$

$r^3 = \frac{5}{\pi}$

$\sin^2 x + \cos^2 x = 1$

b i). $\sec^2 x + \sec x \tan x = \dfrac{1+\sin x}{\cos^2 x}$

LHS $= \sec^2 x + \sec x \tan x$

$= \sec^2 x + \sec x \dfrac{\sin x}{\cos x}$

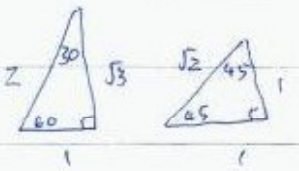
$= \dfrac{1}{\cos^2 x} + \dfrac{1}{\cos x} \times \dfrac{\sin x}{\cos x}$

$= \dfrac{1}{\cos^2 x} + \dfrac{\sin x}{\cos^2 x}$

$= \dfrac{\cos^2 x + \sin x}{\cos^2 x}$

$= \dfrac{1+\sin x}{\cos^2 x}$

$=$ RHS




ii). $\sec^2 x + \sec x \tan x = \dfrac{1}{1-\sin x}$

LHS $= \sec^2 x + \sec x \tan x$

$= \dfrac{1+\sin x}{\cos^2 x}$ from i.

$= \dfrac{1+\sin x}{1-\sin^2 x}$ from $\rightarrow \sin^2 x + \cos^2 x = 1$

$\therefore \cos^2 x = 1 - \sin^2 x$

$= \dfrac{(1+\sin x)}{(1+\sin x)(1-\sin x)}$

$= \dfrac{1}{1-\sin x}$

iii). $\displaystyle\int_0^{\pi/4} \frac{1}{1-\sin x}\, dx$

$= \displaystyle\int_0^{\pi/4} \sec^2 x + \sec x \tan x \, dx$

$= \left[\tan x + \sec x\right]_0^{\pi/4}$

$= \left[\left(\tan\frac{\pi}{4} + \sec\frac{\pi}{4}\right) - (\tan 0 + \sec 0)\right]$

$= \left[(1 + \sqrt{2}) - 0\right]$

$= \sqrt{2}$

$\cos 45 = \frac{1}{\sqrt{2}}$

$\cos = \frac{A}{H}$

$\frac{1}{\cos} = \frac{H}{A}$

$\sec = \sqrt{2}$

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c). $2 = \int_a^1 \frac{1}{x}\,dx + \int_1^b \frac{1}{x}\,dx$

$2 = [\ln x]_a^1 + [\ln x]_1^b$

$2 = [(\ln 1 - \ln a)] + [\ln b - \ln 1]$

$2 = \ln a + \ln b$

$A_1 = 1$

$A_2 = 1$

$\therefore A_1 + A_2 = 2$

You may ask for an extra Writing Booklet if you need more space to answer question 5.