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(a).

(i). $A = 2\pi r^2 + 2\pi r h$.

$V = \pi r^2 h$.

$10 = \pi r^2 h$.

$h = \frac{10}{\pi r^2}$.

Sub into A.

$A = 2\pi r^2 + 2\cancel{\pi} \cancel{r} \left(\frac{10}{\cancel{\pi} r^{\cancel{2}}}\right)$.

$A = 2\pi r^2 + \frac{20}{r}$, as required

(ii). $A = 2\pi r^2 + \frac{20}{r}$.

$\frac{dA}{dr} = 4\pi r \cancel{+} - \frac{20}{r^2}$.

let $\frac{dA}{dr} = 0$.

$4\pi r = \frac{20}{r^2}$.

$4\pi r^3 = 20$.

$\pi r^3 = 5$.

$r^3 = \frac{5}{\pi}$.

$r = \sqrt{\frac{5}{\pi}}$.

$r = 1.261566261$

$r = 1.26$ (2dp).

r	1	1.26	2
$\frac{dA}{dr}$	-7.4	0	20.13...
	\	—	/

$\therefore$ $r = 1.26$ is a minimum.

(b) i $\sec^2 x + \sec x \tan x = \dfrac{1+\sin x}{\cos^2 x}$.

LHS $= \sec^2 x + \sec x \tan x$

$= \dfrac{1}{\cos^2 x} + \dfrac{1}{\cos x} \cdot \tan x$.

$= \dfrac{1}{\cos^2 x} + \dfrac{\frac{\sin x}{\cos x}}{\cos x}$.

$= \dfrac{1}{\cos^2 x} + \dfrac{\sin x}{\cos^2 x}$

$= \dfrac{1+\sin x}{\cos^2 x}$

$=$ RHS.

(ii). $\sec^2 x + \sec x \tan x = \dfrac{1}{1-\sin x}$.

$\dfrac{1+\sin x}{\cos^2 x} = \dfrac{1}{1-\sin x}$. $=$ ~~$\dfrac{1+\sin x}{\cos^2 x}$~~.

~~LHS $= \dfrac{1+\sin x}{\cos^2 x}$~~

~~$= \dfrac{1+\sin x}{1-\sin^2 x}$.~~

$= \dfrac{1+\sin x}{\cos^2 x}$

$= \dfrac{1+\cancel{\sin x}}{(1+\cancel{\sin x})(1-\sin x)}$

$= \dfrac{1}{1-\sin x}$, as required

~~$= \dfrac{1+\sin x}{\cos^2 x}$~~

~~$= \dfrac{1}{\cos^2 x} + \dfrac{\sin x}{\cos^2 x}$.~~

~~$= \sec^2 \theta + \dfrac{\sin x}{\cos^2 x}$~~

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(iii).

$$\cancel{\int_0^{\frac{\pi}{4}} \frac{1}{1-\sin x}\, dx.}$$

$$\cancel{= \left[\frac{1}{1}\tan^{-1}\frac{\sin x}{1}\right]_0^{\frac{\pi}{4}}}$$

$$\cancel{= \left[\tan^{-1}(\sin x)\right]_0^{\frac{\pi}{4}}}$$

$$\cancel{= \left(\tan^{-1}\left(\sin\tfrac{\pi}{4}\right)\right) - \left(\tan^{-1}(\sin 0)\right)}$$

$$\cancel{= \tan^{-1}\frac{1}{\sqrt{2}} - 0}$$

$$\cancel{= \tan^{-1}\frac{1}{\sqrt{2}}}$$

$$\int_0^{\frac{\pi}{4}} \sec^2 x + \sec x \tan x$$

$$= \left[\tan x + \sec x\right]_0^{\frac{\pi}{4}}$$

$$= 1 + \sqrt{2} + 1$$

$$= 2 + \sqrt{2}.$$

(c). $y = \frac{1}{x}$.

$$A_1 \int_a^1 \frac{1}{x}\, dx.$$

$$= \left[\ln x\right]_a^1$$

$$= \ln 1 - \ln a.$$

$$= -\ln a.$$

$$A_2 = \int_1^b \frac{1}{x}\, dx.$$

$$= \left[\ln x\right]_1^b$$

$$= \ln b - \ln \cancel{1}$$

$$= \ln b.$$

Start here.

area of ~~Δ~~ OPTQ

$= 2 \times \Delta OPT.$

$= 2\left(\frac{1}{2} \times 5 \times \sqrt{56}\right).$

$= 5\sqrt{56}.$

$= 10\sqrt{14}.$

Area of Shaded region $= A_{OPTQ} - A_{OPQ}$

$= 10\sqrt{14} - 22.5$

$= 14.9165\ 7387$

$= 14.12$ units^2 (2dp).

Start here.

$\ln b - \ln a = 1$

$b - a = e^1$ $= \log_1 \frac{b}{a}$

$\boxed{b = e^1 + a}$

~~substitute~~

$\ln(e^1 + a) - \ln a = 1$

$1 + \ln a + \ln a = 1$

$\boxed{a = b - e^1}$

$\ln b - \ln a = 1.$

$\ln \frac{b}{a} = 1.$