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(a) i. $V = \pi r^2 h$

$\pi r^2 h = 10$

$\pi r^2 = 10h$ (i)

If $A = 2\pi r^2 + 2\pi r h$ (ii)

sub (i) into (ii)

$A = 2 \times 10h + 2\pi r h$

$\pi r^2 h = 10$

$\pi r^2 = \frac{10}{h}$

$= \frac{1}{h} \times 10$

$\frac{\pi r^2}{10} = \frac{1}{h}$

ii $A = 2\pi r^2 + \frac{20}{r}$

$\frac{dA}{dr} = 4\pi r + \left(-\frac{20}{r^2}\right)$

$= 4\pi r - \frac{20}{r^2}$

$\frac{d^2A}{dr^2} = 4\pi + \frac{40}{r^3}$

$\frac{d^2A}{dr^2} > 0$ for min

Solve: $4\pi + \frac{40}{r^3} \geq 0$

$\frac{40}{r^3} \geq -4\pi$

$r^3 \leq \frac{40}{-4\pi}$

$r \leq -1.471013672$

(b) i. $\sec^2 x + \sec x + \tan x = \dfrac{1 + \sin x}{\cos^2 x}$

$\text{LHS} = \sec^2 x + \sec x + \tan x$

$= \tan^2 x + 1 + \sec x + \tan x$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

ii. $\dfrac{1 + \sin x}{\cos^2 x} = \dfrac{1}{1 - \sin x}$

$\text{LHS} = \dfrac{1 + \sin x}{\cos^2 x}$

$= \dfrac{1 + \sin x}{1 - \sin^2 x}$

$= \dfrac{1}{1 - \sin x}$

$= \text{RHS}$ (as required)

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(b) iii. $\int_0^{\frac{\pi}{4}} \frac{1}{1-\sin x}\,dx =$

$=$

(c) $y = \frac{1}{x}$

$\int_a^1 \frac{1}{x}\,dx = \left[\ln x\right]_a^1$

$= \ln 1 - \ln a$

$\therefore \ln 1 - \ln a = 1$

$\ln\left(\frac{1}{a}\right) = 1$

$\int_1^b \frac{1}{x}\,dx = \left[\ln x\right]_1^b$

$= \ln b - \ln 1$

$\ln b - \ln 1 = 1$

$\ln b = 1$

$\therefore \ln b = \ln\left(\frac{1}{a}\right) = 1$

You may ask for an extra Writing Booklet if you need more space to answer question 5.