

Start here for
Question Number: **5**

a) $V = 10$ cubic m

$$\begin{aligned} \text{i- } A &= 2\pi r^2 + 2\pi r h \\ &= 2\pi r^2 + 2\pi r 10 \\ &= 2\pi r^2 + \pi r 20 \\ &= \underline{2\pi r^2} + \frac{20}{r} \end{aligned}$$

ii- $f(x) = 2\pi r^2 + \frac{20}{r}$

$$f'(x) = 4\pi r + \left(-\frac{1}{2} 20r^{-\frac{3}{2}}\right)$$

minimum occurs when $f'(x) = 0$

$$0 = 4\pi r + \left(-\frac{1}{2} 20r^{-\frac{3}{2}}\right)$$

$$= 4\pi r - 10r^{-\frac{1}{2}}$$

$$= 4\pi r - \sqrt{10r}$$

$$= r(4\pi - \sqrt{10}) \quad \therefore r = 0$$

\therefore minimum occurs when $r = 0$

b) Prove $\sec^2 x + \sec x \tan x = \frac{1 + \sin x}{\cos^2 x}$

$$\frac{1 + \sin x}{\cos^2 x} = \frac{1}{\cos^2 x} + \frac{\sin x}{\cos x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\therefore \sec^2 x + \sec x \tan x = \frac{1}{\cos^2 x} + \tan x$$

$$= \sec^2 x (\sec x + \tan x)$$

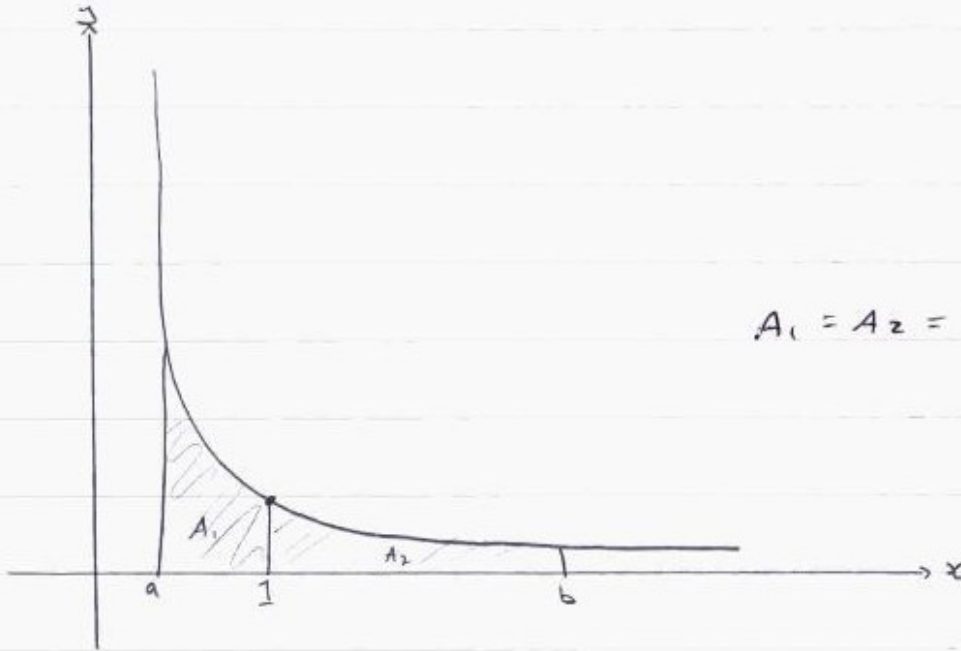
$$= \underline{\underline{\sec^2 x + \sec x \tan x}}$$

$$\begin{aligned}
 \text{ii - prove } \sec^2 x + \sec x \tan x &= \frac{1}{1 - \sin x} \\
 &= \sec x (\sec x + \tan x) \\
 &= \frac{1}{\cos x} + \tan x \\
 &= \frac{1}{1 - \sin x}
 \end{aligned}$$

$$\begin{aligned}
 \text{iii - } \int_0^{\frac{\pi}{4}} \frac{1}{1 - \sin x} dx \\
 &= \int_0^{\frac{\pi}{4}} \frac{1}{1 - \sin^2 x} dx \\
 &= \left[\tan^{-1} \frac{\cot^2 x}{1} \right] \\
 &= \left[\tan^{-1} \cot^2 x \right] \\
 &= \left[\sin x \right]_0^{\frac{\pi}{4}} \\
 &= \left(\sin \frac{\pi}{4} \right) - \left(\sin 0 \right) \\
 &= \sin \frac{\pi}{4} \\
 &= \frac{1}{\sqrt{2}}
 \end{aligned}$$

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$$c) \quad y = \frac{1}{x} \quad x > 0$$



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$$a = \frac{1}{\sqrt{2}}$$

$$b = \sqrt{2}$$

