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(a) (i) M is the midpoint

Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-2+12}{2}, \frac{-4+6}{2}\right)$

$= (5, 1)$

∴ Coordinates of M are (5, 1)

(ii) Gradient of BC $= \frac{y_2-y_1}{x_2-x_1}$

$= \frac{8-6}{6-12}$

$= \frac{2}{-6}$

∴ gradient $= -\frac{1}{3}$

(iii) $\triangle ABC$ is similar to $\triangle AMN$ $\frac{2-5}{2-5} = \frac{1}{-3}$

① $\angle CAB = \angle NAM$ (common angle)

② gradient of BC = gradient of MN $= -\frac{1}{3}$

(iv) M(5, 1), N(2, 2), gradient $-\frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{1}{3}(x - 5)$

$3y - 3 = -x + 5$

$x + 3y - 8 = 0$ equation for MN

(v) length of BC  B(12,6) C(6,8)

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(6-12)^2 + (8-6)^2}$

$= \sqrt{(-6)^2 + (2)^2}$

$= \sqrt{36+4}$

length BC $= \sqrt{40}$ or 6.32456.. (by calc)

(vi) Area of OABC = 44 units$^2$

$\therefore$ Area $= \frac{1}{2} \times b \times h$

$44 = \frac{1}{2} \times b \times h$ (where $b = \sqrt{40}$)

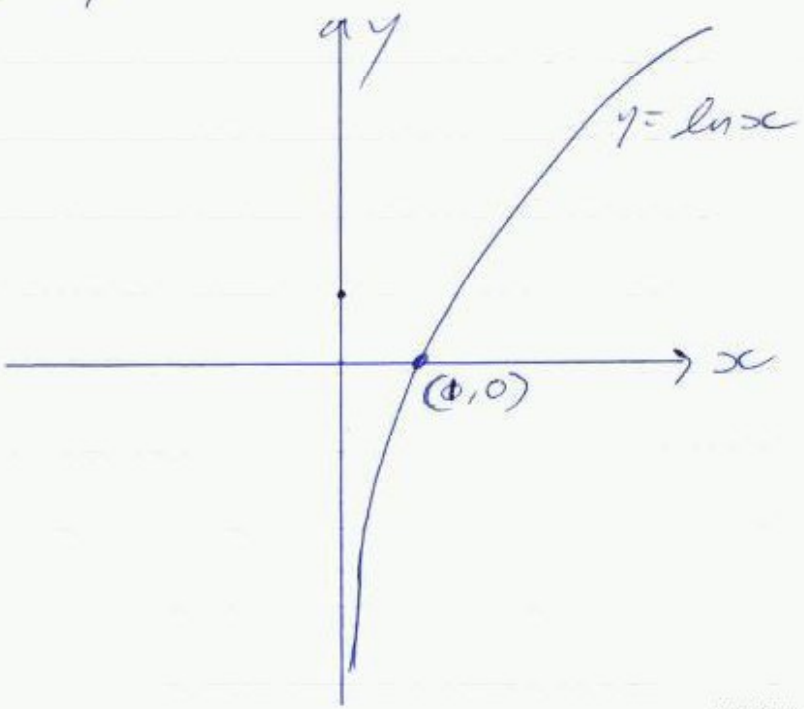
$44 = \frac{1}{2} \times 40 \times h$ ($\div \frac{1}{2}$)

$88 = 40h$

$\therefore h = \frac{88}{40}$

$= 2.2$ units

(b) (i) $y = \ln x$

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(ii) $\int_1^3 \ln x \, dx$ trap rule : $\int_a^b f(x)\,dx$

$= \frac{h}{2}\left[(y_0 + y_n) + 2(y_2 + y_3 + \ldots + y_{n-1})\right]$

$\int_1^3 \ln x \cdot dx = \frac{1}{2}\left[(y_0 + y_3) + 2(y_1 + y_{(2-3)})\right]$

$h = \frac{b-a}{n \to \text{sub intervals}}$

$= \frac{3-1}{2}$

$= 1$

$= \frac{1}{2}\left[(0 + \ln 3) + 2(\ln 2 + \ln 2)\right]$

$= \frac{1}{2}\left[(\ln 3) + 4\ln 2\right]$

$= 1.936$ (3 d.p.)

(iii) The value from part (ii) is greater than the exact value because by using the trapezoidal rule, the value given is only an approximation and there is a margin of error when compared to the exact area.

You may ask for an extra Writing Booklet if you need more space to answer question 3.