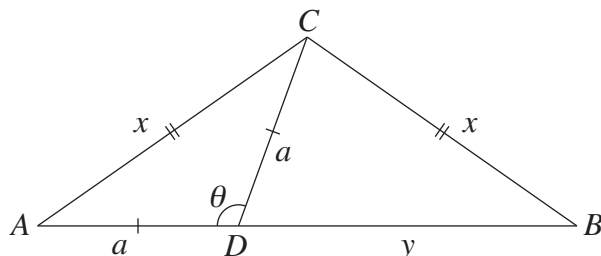


Question 10 (12 marks) Use the Question 10 Writing Booklet.

- (a) In the diagram ABC is an isosceles triangle with $AC = BC = x$. The point D on the interval AB is chosen so that $AD = CD$. Let $AD = a$, $DB = y$ and $\angle ADC = \theta$.

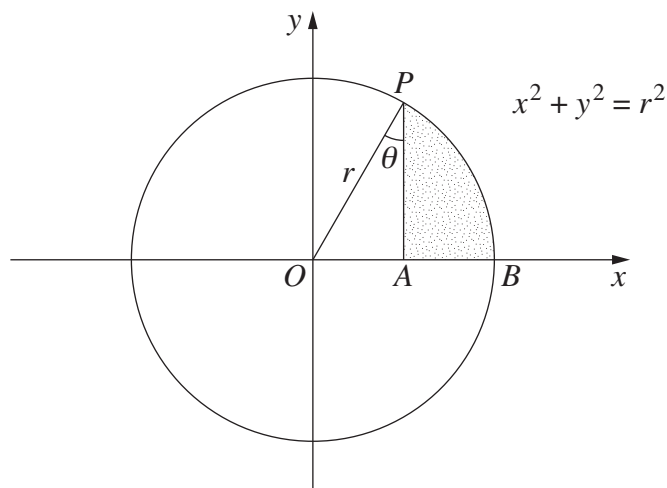


- (i) Show that $\triangle ABC$ is similar to $\triangle ACD$. 2
- (ii) Show that $x^2 = a^2 + ay$. 1
- (iii) Show that $y = a(1 - 2 \cos \theta)$. 2
- (iv) Deduce that $y \leq 3a$. 1

Question 10 continues on page 17

Question 10 (continued)

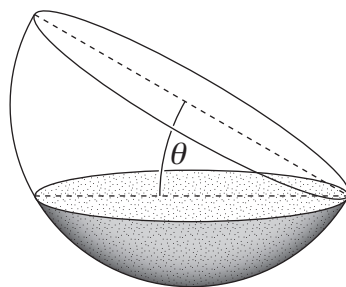
- (b) The circle $x^2 + y^2 = r^2$ has radius r and centre O . The circle meets the positive x -axis at B . The point A is on the interval OB . A vertical line through A meets the circle at P . Let $\theta = \angle OPA$.



- (i) The shaded region bounded by the arc PB and the intervals AB and AP is rotated about the x -axis. Show that the volume, V , formed is given by **3**

$$V = \frac{\pi r^3}{3} (2 - 3\sin\theta + \sin^3\theta).$$

- (ii)



A container is in the shape of a hemisphere of radius r metres. The container is initially horizontal and full of water. The container is then tilted at an angle of θ to the horizontal so that some water spills out.

- (1) Find θ so that the depth of water remaining is one half of the original depth. **1**
- (2) What fraction of the original volume is left in the container? **2**

End of paper