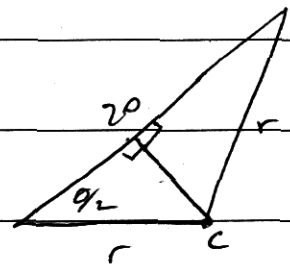
BOARD OF STUDIES
NEW SOUTH WALES

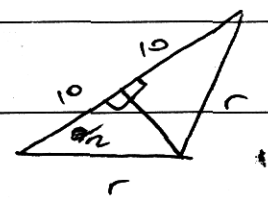
Question 10

a) i).



— two triangles are isosceles → (S.S.S.)

∴ diagram becomes



View notes

$$\therefore \cos\frac{\theta}{2} = \frac{10}{r}$$

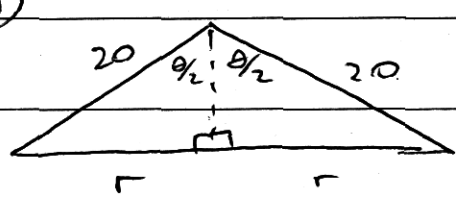
$$r\cos\frac{\theta}{2} = 10$$

$$r = \frac{10}{\cos\theta/2}$$

S	A
T	C

$\therefore r = 10\sec\frac{\theta}{2}$ for $0 < \theta \leq \pi/2$

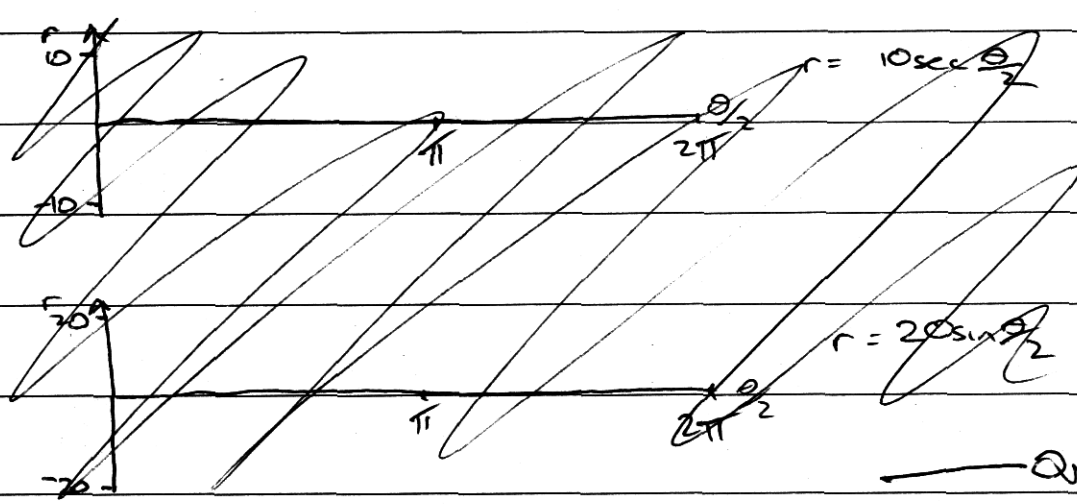
ii)



$$\sin\frac{\theta}{2} = \frac{r}{20}$$

$$\therefore 20\sin\frac{\theta}{2} = r \quad (\theta \text{ is obtuse}).$$

iii)



— Over page.

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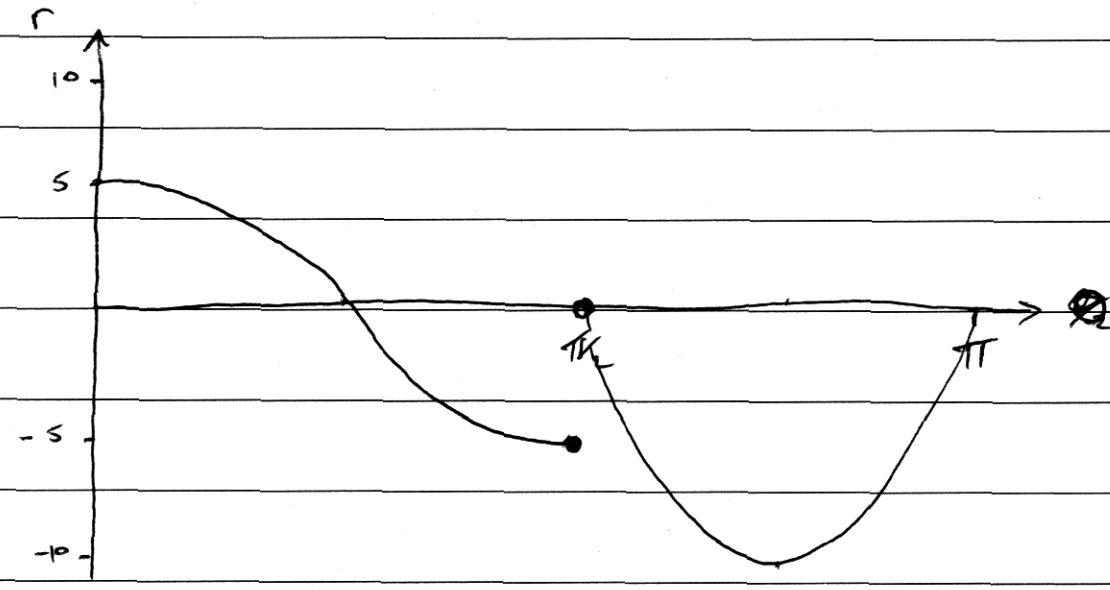
iii)



$r = 10\sin\theta \rightarrow \pi/2 < \theta \leq \pi$

$r = 5\sec\theta \rightarrow 0 < \theta \leq \pi/2$

b) $\dfrac{dl}{dx} = \dfrac{-1.(2x+16)}{\left[b^2+(x+8)^2\right]^2} + \dfrac{-1.(2x-16)}{\left[b^2+(x-8)^2\right]^2}$

$= \dfrac{-2x-16}{\left[b^2+(x+8)^2\right]^2} + \dfrac{-2x+16}{\left[b^2+(x-8)^2\right]^2}$

$= \dfrac{-2(x+8)}{\left[b^2+(x+8)^2\right]^2} - \dfrac{2(x-8)}{\left[b^2+(x-8)^2\right]^2}$

$= \dfrac{-2(x+8)\left[b^2+(x-8)^2\right]^2 - 2(x-8)\left[b^2+(x+8)^2\right]^2}{\left[b^2+(x+8)^2\right]^2\left[b^2+(x-8)^2\right]^2}$

common factor $= \dfrac{-2\left((x+8)\left[b^2+(x-8)^2\right]^2 + (x-8)\left[b^2+(x+8)^2\right]^2\right)}{\left[b^2+(x+8)^2\right]^2\left[b^2+(x-8)^2\right]^2}$

$= \dfrac{-2P}{Q}$

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ii) at $x = 0$

$$\frac{dL}{dx} = \frac{-2\left[(x+8)\left(b^2+(x-8)^2\right)^2 + (x-8)\left(b^2+(x+8)^2\right)^2\right]}{\left(b^2+(x+8)^2\right)^2\left(b^2+(x-8)^2\right)^2}$$

test concavity of max. value point.

x	0^-	0	0^+
$\frac{dL}{dx}$	>0	0	<0

positive to right of origin

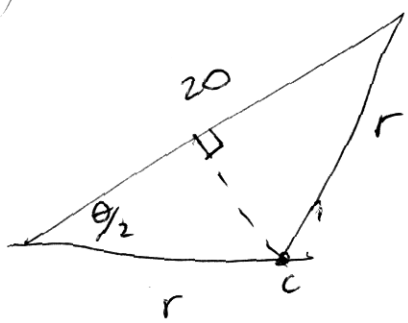
negative to left of origin

∴ a change in concavity occurs.

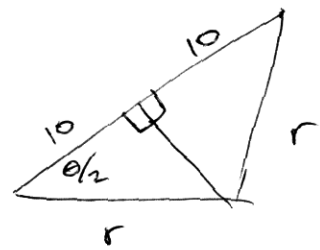
∴ Brightness increases to a maximum at $x=0$, and decreases as P travells towards the right.

iii) The brightness on Hero would change in the same fashion - it would increase until it passes the origin (mid-point), where maximum brightness occurs and would decrease as it travels beyond the origin. The only difference between Hero and Saga is since Hero is closer to the coast, the intensity of light received by Hero is much greater than that received by Saga.

d)



isosceles △

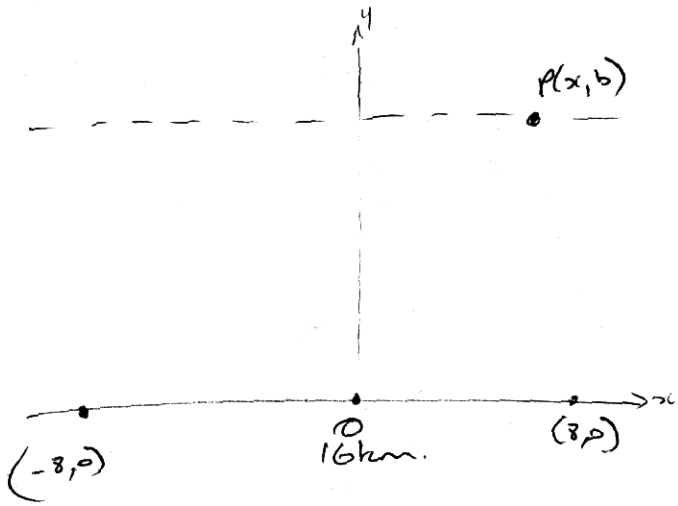


$\therefore \cos \theta/2 = 10/r$

$\therefore r \cos \theta/2 = 10$

$\therefore r = \frac{10}{\cos \theta/2}$

$= 10 \sec \theta/2.$



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