a) (i) $x^2 = 20^2 + r^2 - 2(20)r\cos\frac{\theta}{2}$

$40r\cos\frac{\theta}{2} = 400$

$r\cos\frac{\theta}{2} = 10$

$r = \frac{10}{\cos\frac{\theta}{2}}$

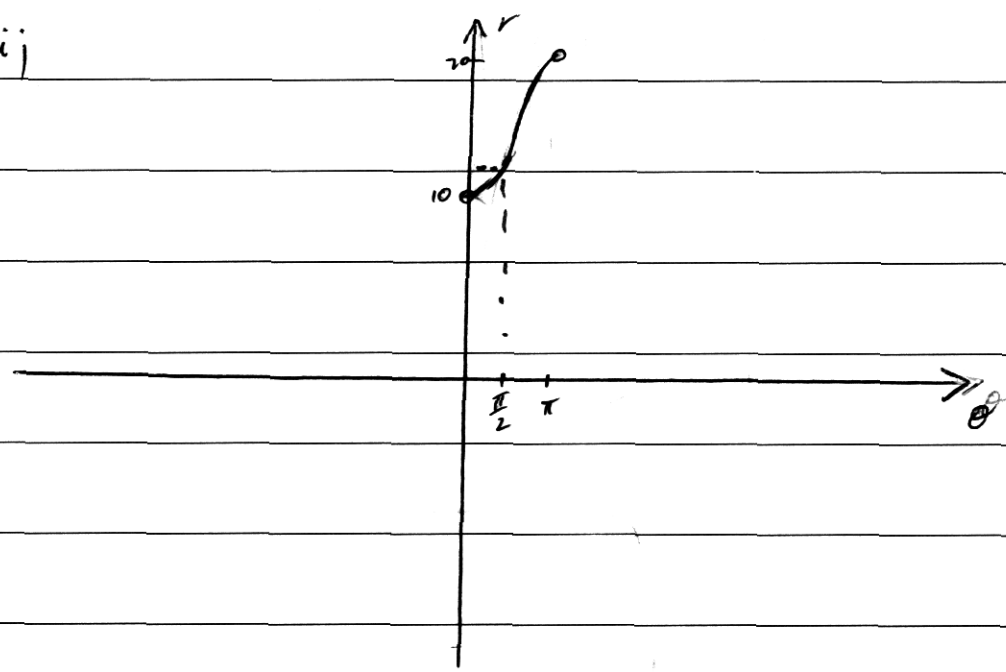
$\therefore r = 10\sec\frac{\theta}{2}$

(ii) $\frac{20}{\sin\frac{\pi}{2}} = \frac{r}{\sin\frac{\theta}{2}}$

$r\sin\frac{\pi}{2} = 20\sin\frac{\theta}{2}$

$\therefore r = 20\sin\frac{\theta}{2}$

(iii)



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6 (i)

$$I = \frac{b^2 + (x-8)^2 + b^2 + (x+8)^2}{\left[b^2 + (x+8)^2\right]\left[b^2 + (x-8)^2\right]}$$

$$\frac{dI}{dx} = \frac{\left[b^2+(x+8)^2\right]\left[b^2+(x-8)^2\right]\left(2(x-8) + 2(x+8)\right)\left(b^2+(x-8)^2+b^2+(x+8)^2\right)}{\left(b^2+(x+8)^2\right)^2\left(b^2+(x-8)^2\right)^2} \times$$

$$\left(2b^2(x-8) + 2b^2(x+8) + 4(x^2-64)\right)$$

$$= \frac{-\left[(x+8)\left(b^2+(x-8)^2\right) + (x-8)\left(b^2+(x+8)^2\right)\right] \times 2}{\left(b^2+(x+8)^2\right)^2\left(b^2+(x-8)^2\right)^2}$$

$$= \frac{-2P}{Q}$$

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(ii')

$P = 2x\left[\left(x^2+64+b^2\right)^2 - \left[256\left(64+b^2\right)\right]\right]$

$= 2x\left(\left(x^2+64+b^2\right)^2 - \left(16384 + 256b^2\right)\right)$

$\frac{dP}{dx} = 2\left(\left(x^2+64+b^2\right)^2 - \left(16384+256b^2\right)\right) + 2(2x)(2x)$

$= 2\left(x^2+64+b^2\right)^2 - 2\left(16384+256b^2\right) + 8x^2$

$= 2\left(\left(x^2+64+b^2\right)^2 - 16384 - 256b^2 + 4x^2\right)$

$= 2\left(x^4 + 2\left(64+b^2\right) + \left(64+b^2\right)^2 - 16384 - 256b^2 + 4x^2\right)$

when $\frac{dP}{dx} = 0$

$\therefore 0 = \left(x^4 + 128 + 2b^2 + 4096 + 128b^2 + b^4 - 16384 - 256b^2 + 4x^2\right)$

$x^4 - 126b^2 + 4x^2 - 12160 = 0$

$x^4 + 4x^2 - 126b^2 - 12160 = 0$

$\sqrt{x^2\left(x^2+4\right)} = \sqrt{12160 + 126b^2}$

$x(x+2) =$

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(ii) P(x, 15)

when P = 15

$$P = 2x\left(\left(x^2+64+b^2\right)^2 - \left(16\sqrt{64+b^2}\right)^2\right)$$

$$= 2x\left(x^4 + 2(64+b^2) + b^4 - 16^2(64+b^2)\right)$$

$$= 2x^5 + 4x(64+b^2) + b^4 - 16384 - 256b^2$$

$$\frac{dP}{dx} = 10x^4 + 256 + 4b^2 = 0$$

$$x^4 = \frac{-4b^2 - 256}{10}$$

but $b = 16$.

$$\therefore x^4 = 128$$

$$x$$

(iii) As Hero sails left to right the brightness increases to a maximum at $x = 0$, then from $x > 0$, the brightness decreases at the same rate that it initially increased.

at $x = 0$, $\frac{d^2I}{dx^2} < 0$ $\therefore$ Max brightness at $x = 0$.

$\therefore$ Concave down curve.