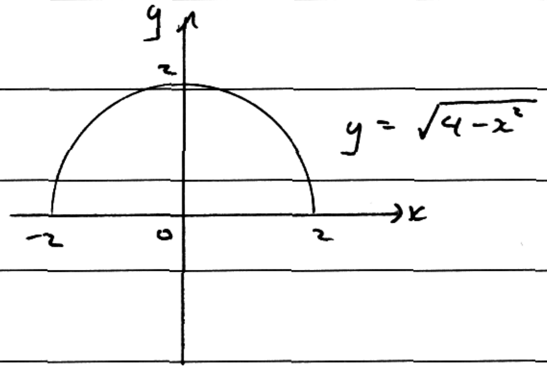
BOARD OF STUDIES NEW SOUTH WALES

(a)



Range $0 \le y \le 2$.

(b) (i) $f'(x) = 3(x+1)(x-3)$

$= 3(x^2 - 2x - 3)$

$= 3x^2 - 6x - 9$

$f(x) = \int 3x^2 - 6x - 9 \, .dx.$

$= x^3 - 3x^2 - 9x + C.$

at $(0, 12)$ $x = 0$ and $f(x) = 12$.

$12 = 0^3 - 3(0)^2 - 9(0) + C$

$C = 12$

$\therefore f(x) = x^3 - 3x^2 - 9x + 12.$

BOARD OF STUDIES
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(b) (ii) $f'(x) = 3x^2 - 6x - 9 = 0$, for stationary pts.

$3(x+1)(x-3) = 0$

$x = -1$ or 3.

$f(-1) = -1^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17.$ $(-1, 17)$

$f(3) = 3^3 - 3(3)^2 - 9(3) + 12$

$= 27 - 27 - 27 + 12.$

$= -15$ $(3, -15)$

$f''(x) = 6x - 6$

$f''(-1) = 6(-1) - 6.$

$= -12 < 0$ $\therefore (-1, 17)$ is max. pt.

$f''(3) = 6(3) - 6$

$= 12 > 0$ $\therefore (3, -15)$ is min. pt.

x	-1^-	-1	-1^+	3^-	3	3^+
$f'(x)$	$-$	0	$-$		0	

$f''(x) = 6x - 6 = 0$, for inflection pts.

$6x = 6$

$x = 1.$

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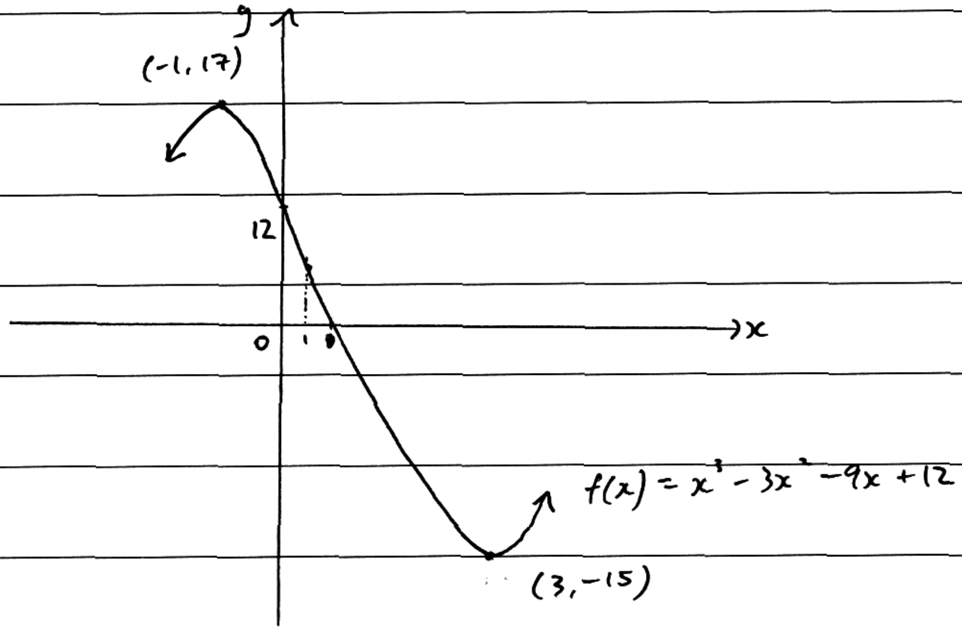
(b) (ii)



(iii) $f''(x) = 6x - 6 = 0$, for inflection pts.

$6x = 6$

$x = 1$.

$\therefore \quad x \geq 1$.

(c) $V_y = \pi \int_0^4 x^2 \, dy$

$= \pi \int_0^4 (4y)^{\frac{1}{2}} \, dy$.

$= \pi \left[\frac{2}{3} 4y^{\frac{3}{2}} \right]_0^4$

$= \frac{2}{3}\pi \left[4(4)^{\frac{3}{2}} \right] - \left[4(0)^{\frac{3}{2}} \right]$

$= \frac{2}{3}\pi \left[4(8) \right] - \left[4(0) \right]$

$= \frac{64\pi}{3}$ units^3

OR

$= 67.02064328$ units^3 (by cal.)