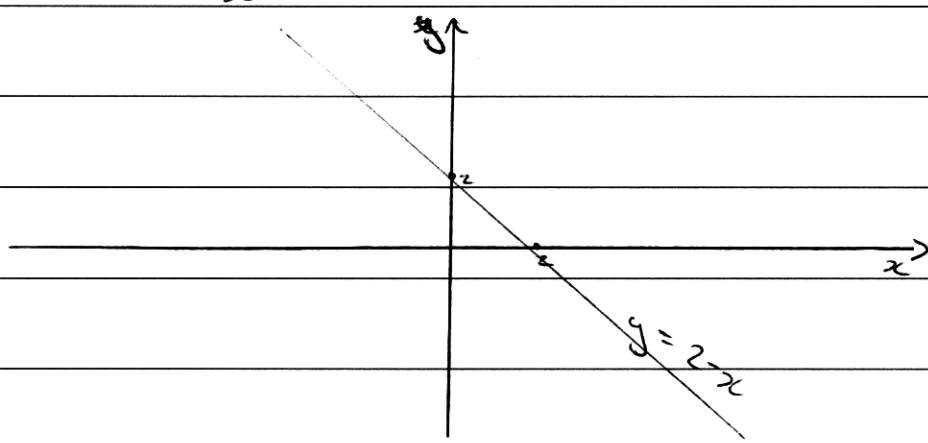
BOARD OF STUDIES
NEW SOUTH WALES

Question 6

a) $y = (4-x^2)^{\frac{1}{2}}$

$= 2 - x$



Rang = y has values for all real values of x

b) $f'(x) = 3(x+1)(x+3)$

$= (3x+3)(x-3)$

$= 3x^2 - 9x + 3x - 9$

$y' = 3x^2 - 6x - 9$

$y = x^3 - 3x^2 - 9x + c$

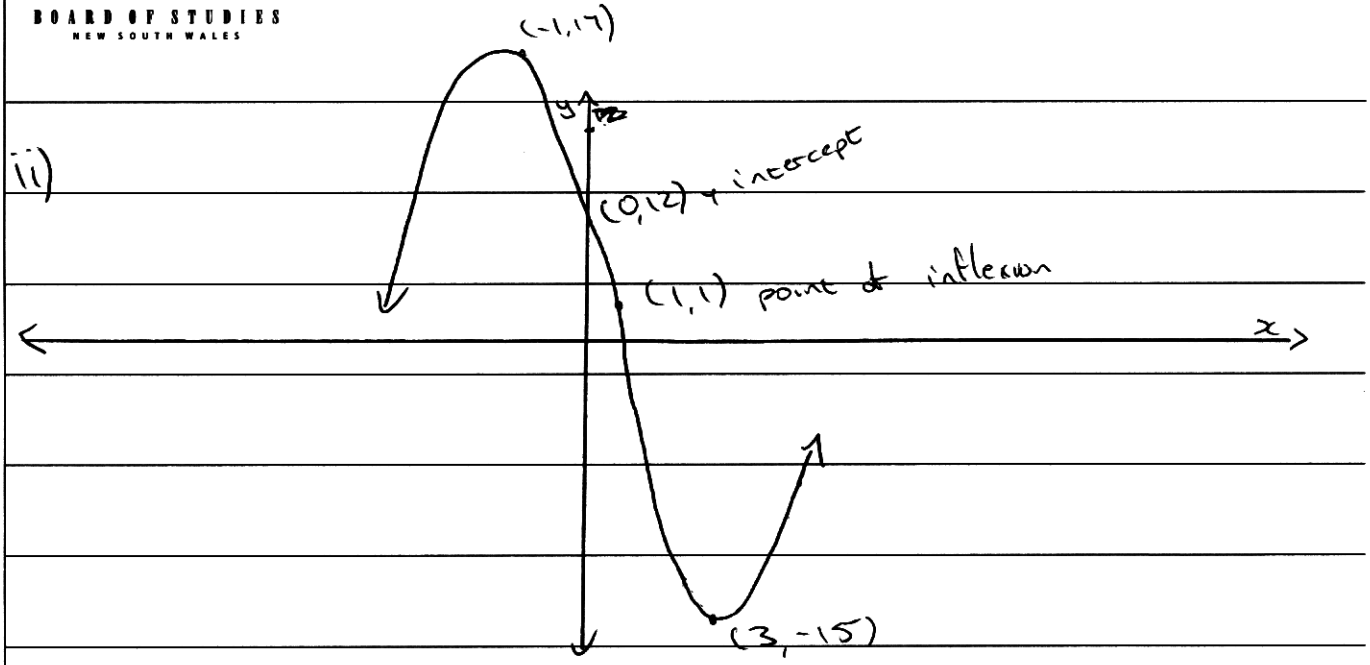
$12 = 0^3 - 3(0)^2 - 9(0) + c$

$c = 12$

$\therefore f(x) = x^3 - 3x^2 - 9x + 12$

BOARD OF STUDIES
NEW SOUTH WALES

ii)



$y = x^3 - 3x^2 - 9x + 12$

$y' = 3x^2 - 6x - 9$

$y'' = 6x - 6$

$(3)^3 - 3(3)^2 - 9(3) + 12$

$27 - 27 - 27 + 12$

$= -15$

stat points occur at $y' = 0$

$\therefore x = -1$ and $x = 3$

test for maxima/minima (sub into y'')

$y'' = 6(-1) - 6 = -12 < 0$ maximum $(-1, 17)$

$y'' = 6(3) - 6 = 12 > 0$ minimum $(3, -15)$

point of inflexion occurs at $y'' = 0$

$0 = 6x - 6$

$x = 1$ $\quad y = 1^3 - 3(1)^2 - 9(1) + 12$

point of inflexion $(1, 1)$ $\quad = 1 - 3 - 9 + 12$

$= 1$

BOARD OF STUDIES
NEW SOUTH WALES

Cuts x axis at $y=0$

$0 = x^3 - 3x^2 - 9x + 12$

~~$-12 = x(x^2 - 3x - 9)$~~

$9x = x^3 - 3x^2 + 12$

$9 = x^2 - 3x + 12$

$(x-3)(x-1)$

iii) $x \geq 1$

c) $\pi \int y^2 dx$ $\quad y = \frac{x^4}{4}$

$4y = x^4$

$\sqrt{4y} = x^2$

$\pi \int_0^4 (4y)^{\frac{1}{2}}$

$\left(\frac{2}{3}(4y)^{\frac{3}{2}}\right)_0^4$

$\pi\left(\frac{2}{3}(4\times4)^{\frac{3}{2}}\right)$

$= 134.04 \text{ u}^3$