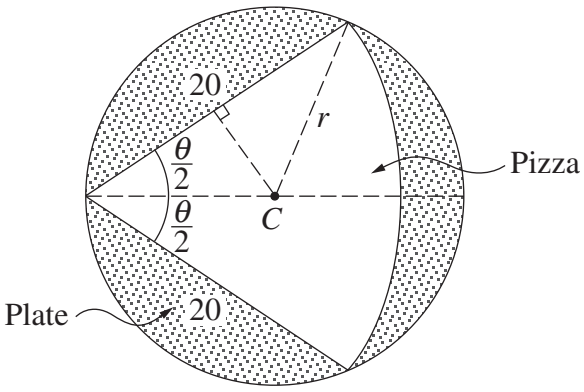
Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a) A circular pizza of radius 20 cm is cut into sectors. Each sector is to be placed on a circular plate that is just large enough to contain that sector.

(i) A sector of pizza is cut where the angle θ at its centre satisfies $0 < \theta \leq \frac{\pi}{2}$. **2**

It is placed on a circular plate, of radius r cm and centre C, as shown below.



Show that $r = 10\sec\frac{\theta}{2}$ for $0 < \theta \leq \frac{\pi}{2}$.

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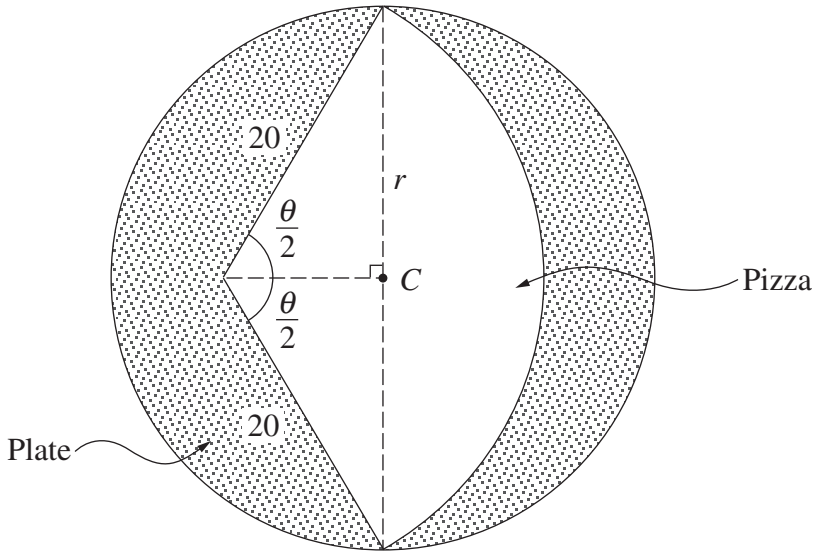
Marks

Question 10 (continued)

(ii) Another sector of pizza is cut where the angle θ at its centre satisfies **1**

$$\frac{\pi}{2} < \theta < \pi .$$

This sector of pizza is placed on a circular plate as shown below. Again, we let the radius of the plate be r cm, and we let the centre be C.



Show that $r = 20\sin\frac{\theta}{2}$ for $\frac{\pi}{2} < \theta < \pi$.

(iii) Sketch the graph of r, as defined by the equations in parts (i) and (ii), for $0 < \theta < \pi$. **3**

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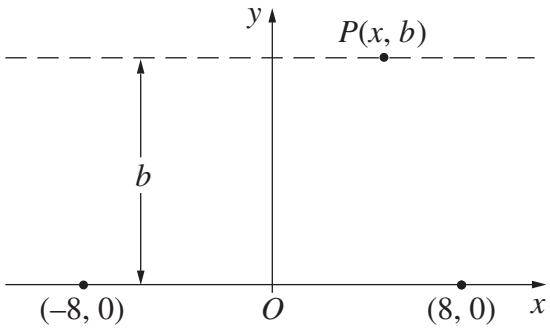
Marks

Question 10 (continued)

(b) On a dark night, two ships, Saga and Hero, sail parallel to a straight coastline on which there are two lights of equal brightness, 16 kilometres apart.



Suppose the coastline is represented by the x axis where the origin O is chosen to be the midpoint of the light sources. It is known that the (total) brightness from the lights on a ship at point $P(x, b)$ is

$$I = \frac{1}{b^2 + (x+8)^2} + \frac{1}{b^2 + (x-8)^2} \ .$$

(i) Show that $\dfrac{dI}{dx} = -\dfrac{2P}{Q}$ where **2**

$$P = \left[(x+8)\left(b^2 + (x-8)^2\right)^2 + (x-8)\left(b^2 + (x+8)^2\right)^2 \right]$$

and $Q = \left(b^2 + (x+8)^2\right)^2 \left(b^2 + (x-8)^2\right)^2$.

To answer parts (ii) and (iii), you may assume the following factorisation, given by a computer package, that

$$P = 2x\left(x^2 + 64 + b^2 + 16\sqrt{64 + b^2}\right)\left(x^2 + 64 + b^2 - 16\sqrt{64 + b^2}\right).$$

(ii) Saga sails parallel to the coast at a distance 15 km from the coast. **2**

By considering $\dfrac{dI}{dx}$, show that, as Saga sails from left to right, the brightness on Saga increases to a maximum when $x = 0$ and then decreases.

(iii) Hero sails parallel to the coast at a distance 6 km from the coast. **2**

Describe how the brightness on Hero changes as Hero sails from left to right. Give clear reasons for your answer.

End of paper