Question 6

a) (i) ~~$T_n = \frac{n}{2}[a + (n-1)r]$~~

~~$T_{60} = \frac{1}{2}[\ldots(59)]$~~

$$T_n = a + (n-1)r$$

$$T_{60} = -1 + (59)5$$

$$= -1 + 295$$

$$= 294$$

$\therefore$ The 60th term is 294.

(ii) $S_n = a + (r - l)d$

c) $y = x^3 + x^2 - x + 2$

(i) A and B are turning pts and hence, $y' = 0$ at both. Since A is a max. turning pt. $y'' < 0$ at A and since B is a min. turning pt. $y'' > 0$ at B.

$y' = 3x^2 + 2x - 1$

For $y' = 0$

$3x^2 + 2x - 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-2 \pm \sqrt{4 - 4 \times 3 \times -1}}{6}$

$= \dfrac{-2 \pm \sqrt{4 + 12}}{6}$

$= \dfrac{-2 \pm \sqrt{16}}{6}$

$= \frac{-2+4}{6}$

$= \frac{2}{6}$

sub $\frac{2}{6}$ into eqn. to find pt.

$y = \left(\frac{2}{6}\right)^3 + \left(\frac{2}{6}\right)^2 - \frac{2}{6} + 2$

$= \frac{8}{216} + \frac{4}{36} - \frac{2}{6} + 2$

$= \frac{8}{216} + \frac{24}{216} - \frac{72}{216} + 2$

$= \cancel{\frac{104}{216}} - \frac{40}{216} + 2$

$B\left(\frac{1}{3}, 1\frac{176}{216}\right)$

$\therefore$ is a turning pt. This would be pt B. as shown on the diagram.

For $\frac{-2-4}{6}$

$= -\frac{6}{6}$

$\therefore$ a turning occurs when $x = -1$

sub $x = -1$ into y.

$y = (-1)^3 + (-1)^2 - (-1) + 2$

$= -1 + 1 + 1 + 2$

$= 3$

$\therefore$ a turning pt. also occurs at $(-1, 3)$
$\therefore$ pt $A(-1, 3)$ as shown roughly on the diagram.

(ii) As pt $B\left(\frac{1}{3}, 1\frac{176}{216}\right)$ is a minimum, the curve is concave up for $x > \frac{1}{3}$

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(iii) $x^3 + x^2 - x + 2 = k$